**Enrollment No: ____________________** **Exam Seat No:____________________**

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

**Subject Name : Engineering Mathematics - III**

**Subject Code : 4TE03EMT2** **Branch: B. Tech (All)**

**Semester : 3** **Date : 27/11/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** One of the Dirichlet's condition is function f(x) should be
(A) single valued (B) multi valued (C) real valued (D) None of these

**b)** If $f(x)=x^2$ is represented by Fourier series in $(-\pi,\ \pi)$ then $b_n$ equal to
(A) $\pi^2/3$ (B) 0 (C) $2\pi^2/3$ (D) $\pi^2/6$

**c)** In the Fourier series expansion of $f(x)=x^3$ in $(-1, 1)$
(A) only sine terms are present (B) both sine and cosine terms are present
(C) only cosine terms are present (D) constant term is present

**d)** Laplace transform of $e^{2t+3}$ is
(A) $\frac{e^3}{s-2}(s>2)$ (B) $\frac{e^2}{s-3}(s>3)$ (C) $\frac{1}{s-\log 2}$ (D) $\frac{1}{s-2}$

**e)** Laplace transform of $\frac{\sin t}{t}$ is
(A) $\cot^{-1}\frac{1}{s}$ (B) $\tan^{-1}s$ (C) $\tan^{-1}\frac{1}{s}$ (D) $\sin^{-1}s$

**f)** Inverse Laplace transform of $\frac{12}{s^2-9}$ is
(A) $3\sinh 4t$ (B) $4\sinh 3t$ (C) $4\cosh 3t$ (D) $3\cosh 4t$

**g)** $\frac{1}{D-a}X$ , (Where $X=k$ is constant) equal to
(A) $-\frac{k}{a}$ (B) $\frac{k}{a}$ (C) $ka$ (D) $-ka$

**h)** The C. F. of the differential equation $\left(D^2-3D+2\right)y=e^{2x}$ is
(A) $c_1e^{x}+c_2e^{2x}$ (B) $c_1e^{-x}+c_2e^{-2x}$ (C) $c_1e^{-x}+c_2e^{2x}$ (D) $c_1e^{x}+c_2e^{-2x}$

**i)** The P. I of $\left(D^2+6D+5\right)y=4e^{-x}$ is
(A) $4xe^{-x}$ (B) $4xe^{x}$ (C) $xe^{x}$ (D) $xe^{-x}$

**j)** Eliminating arbitrary function from $z = f\left(x^2+y^2\right)$, the partial differential equation formed is

(A) $xq = yp$ (B) $xp = yq$ (C) $z = pq$ (D) None of these

**k)** The general solution of the equation $xp + yq = z$ is

(A) $F\left(\frac{x}{y}, \frac{y}{z}\right)=0$ (B) $F\left(xy, x+y\right)=0$ (C) $F\left(\frac{y}{x}, \frac{z}{y}\right)=0$

(D) None of these

**l)** The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(A) $z = f_1(y+x) + f_1(y-x)$ (B) $z = f_1(y+x) + f_2(y-x)$

(C) $z = f_2(y+x) + f_2(y-x)$ (D) $z = f\left(x^2 - y^2\right)$

**m)** The order of convergence in Bisection method is

(A) linear (B) quadratic (C) zero (D) None of these

**n)** Iterative formula for finding the square root of N by Newton-Raphson method is

(A) $x_{i+1} = \frac{1}{2}\left(x_i - \frac{N}{x_i}\right)$ $(i = 0,1,2,.....)$ (B) $x_{i+!} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$ $(i = 0,1,2,.....)$

(C) $x_{i+1} = x_i\left(2 - Nx_i\right)$ $(i = 0,1,2,.....)$ (D) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** Using Newton-Raphson method, find the root of $f(x) = \sin x + \cos x$ correct to three decimal places. **(5)**

**b)** One real root of the equation $x^3 - 4x - 9 = 0$ lies between 2.625 and 2.75. Find the root using Bisection method. **(5)**

**c)** Evaluate: $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$ **(4)**

**Q-3** **Attempt all questions** **(14)**

**a)** Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}$ in the interval $-\pi \le x \le \pi$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ..... = \frac{\pi^2}{12}$. **(5)**

**b)** Obtain Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$ **(5)**

**c)** One real root of the equation $x^3 - 4x + 1 = 0$ lies between 1 and 2. Find the root correct to three significant digits using Secant method. **(4)**

**Q-4** **Attempt all questions** **(14)**

**a)** Using Laplace transform method solve: **(5)**

$y'' + 3y' + 2y = e^t, \quad y(0) = 1, \quad y'(0) = 0$

**b)** Evaluate: $L^{-1}\left(\frac{s}{s^4 + s^2 + 1}\right)$ **(5)**

**c)** Solve: $pz - qz = z^2 + (x+y)^2$ **(4)**

**Q-5** **Attempt all questions** **(14)**

**a)** Using convolution theorem evaluate: $L^{-1}\left(\dfrac{s}{(s^2+4)^2}\right)$ **(5)**

**b)** Solve: $\dfrac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}$ **(5)**

**c)** Solve: $\dfrac{\partial^2 z}{\partial x^2} - 4\dfrac{\partial^2 z}{\partial y^2} = \cos 2x \ \cos 2y$ **(4)**

**Q-6** **Attempt all questions** **(14)**

**a)** Solve: $D^2(D^2+4)y = 48x^2$ **(5)**

**b)** Obtain a half – range sine series to represent $f(x) = lx - x^2$ in the range $(0,\ l)$ . **(5)**

**c)** Evaluate: $L(t\ e^{2t} \cos 3t)$ **(4)**

**Q-7** **Attempt all questions** **(14)**

**a)** Using the method of variation of parameters, Solve: $y'' + a^2 y = \sec ax$ **(5)**

**b)** Solve: $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 2\cos[\log(1+x)]$ **(5)**

**c)** Solve: $\dfrac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$, $\dfrac{\partial z}{\partial x} = a \sin y$ and $\dfrac{\partial z}{\partial y} = 0$. **(4)**

**Q-8** **Attempt all questions** **(14)**

**a)** Determine the Fourier series up to and including the second harmonic to represent the periodic function $y = f(x)$ defined by the table of values given below. $f(x) = f(x+2\pi)$ **(7)**

| $x^\circ$ | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 0.5 | 0.8 | 1.4 | 2.0 | 1.9 | 1.4 | 1.2 | 1.4 | 1.1 | 0.5 | 0.3 | 0.4 |

**b)** Using the method of separation of variables, **(7)**

solve $\dfrac{\partial \mathrm{u}}{\partial \mathrm{x}} = 2\dfrac{\partial \mathrm{u}}{\partial \mathrm{t}} + \mathrm{u}$, given $\mathrm{u}(\mathrm{x},\ 0) = 6e^{-3\mathrm{x}}$